Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Wednesday 5 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **6668/01**

Further Pure Mathematics FP2

Advanced/Advanced Subsidiary

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{2x}$$

(6)

Question 1 continued

Q1

(Total 6 marks)

2. Use algebra to find the set of values of x for which

$$\frac{6}{x-1} < \frac{x+6}{x}$$

(6)

Question 2 continued

Question 2 continued

Question 2 continued

Q2

(Total 6 marks)

3. (a) Express $\dfrac{2}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Hence find, in terms of n,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} \qquad n \in \mathbb{N}, n > 1$$

Give your answer in the form $\dfrac{n(n+A)}{B(n+1)(n+2)}$, where A and B are constants to be found.

(4)

Question 3 continued

Question 3 continued

Question 3 continued

Q3

(Total 7 marks)

4. A transformation T from the z-plane, where $z = x + iy$, to the w-plane, where $w = u + iv$, is given by

$$w = z^2 + 4$$

The line in the z-plane with equation $y = 2$ is mapped by T onto the curve C in the w-plane.

(i) Show that C is a parabola.

(ii) Find the coordinates of the focus of C.

(iii) Find an equation for the directrix of C.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

Q4

(Total 6 marks)

5. Given that

$$y\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^2 - 5y = 0$$

(a) find an expression for $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y.

(4)

Given also that $y = 4$ and $\frac{dy}{dx} = \frac{1}{2}$ at $x = 0$

(b) find a series solution for y in ascending powers of x with simplified coefficients, up to and including the term in x^3

(4)

Question 5 continued

Question 5 continued

Question 5 continued

Q5

(Total 8 marks)

6. (a) Show that the substitution $v = y^{-3}$ transforms the differential equation

$$x\frac{dy}{dx} + 2y = 3x^4y^4 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} - \frac{6}{x}v = -9x^3 \qquad x > 0 \qquad \text{(II)}$$

(5)

(b) Find the general solution of the differential equation (II).

(5)

(c) Hence obtain the general solution of the differential equation (I), giving your answer in the form $y^3 = f(x)$.

(1)

Question 6 continued

Question 6 continued

Question 6 continued

Q6

(Total 11 marks)

7. (a) Use de Moivre's theorem to show that

$$\sin 5\theta - 5\sin\theta \equiv 16\sin^5\theta - 20\sin^3\theta$$

(5)

(b) Hence find the 5 distinct solutions of the equation

$$32x^5 - 40x^3 + 10x - 1 = 0$$

giving your answers to 3 decimal places where appropriate.

(5)

(c) (i) Find $\int (8\sin^5\theta - 10\sin^3\theta)\,d\theta$

(ii) Hence find $\int_0^{\frac{\pi}{3}} (8\sin^5\theta - 10\sin^3\theta)\,d\theta$

(4)

Question 7 continued

Question 7 continued

Question 7 continued

Q7

(Total 14 marks)

8.

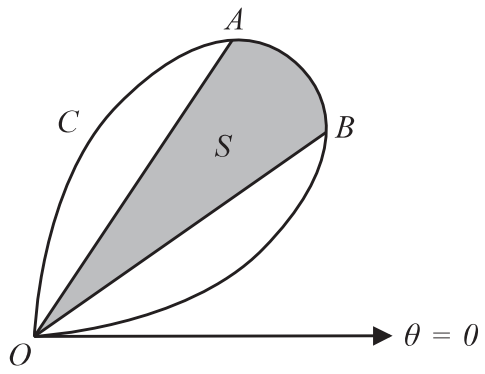

Figure 1

Figure 1 shows a curve C with polar equation

$$r = a\sin 2\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

where a is a positive constant.

The point A has polar coordinates (R, ϕ). The tangent to C at A is parallel to the initial line.

(a) Show that $\tan\phi = \sqrt{2}$

(4)

(b) Find, in terms of a, the exact value of R.

(2)

The tangent to C at B is perpendicular to the initial line. The region S, shown shaded in Figure 1, is bounded by OA, OB and C, where O is the pole.

(c) Show that the area of S is given by

$$\frac{1}{2}a^2\int_{\arctan\left(\frac{1}{\sqrt{2}}\right)}^{\arctan\sqrt{2}}\frac{1}{2}(1-\cos 4\theta)\,\mathrm{d}\theta$$

(5)

(d) Hence show that the exact area of S is

$$a^2\left(\frac{\sqrt{2}}{18}-\frac{\pi}{8}+\frac{1}{2}\arctan\sqrt{2}\right)$$

(6)

Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

Q8

(Total 17 marks)

TOTAL FOR PAPER: 75 MARKS

END